

Vortex in axion condensate as a dark matter halo

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We study vortices in axion condensate on the galactic scale. Such vortices can occur as a result of global rotation of the early universe. We study analytical models of vortices and calculate exemplary galaxy rotation curves. Depending on the setup it is possible to obtain a vast variety of shapes which give a good qualitative agreement with observational results.

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I. INTRODUCTION

The problem of dark matter seems to be a challenge for theoretical and observational investigations. Recent astronomical observations, like the measurements of cosmic microwave background (CMB) radiation by WMAP [1], or measurements of distant supernova type Ia (SNIa) [2, 3], indicate that the Universe, apart from matter, is fulfilled with a phenomenological fluid with negative pressure (called dark energy) which could be responsible for the current acceleration of the universe. While the nature of this energy is still unknown (the cosmological constant Λ is the most serious candidate), the combination of results from CMB measurements, SNIa data and extragalactic observations indicate that as much as 2/3 of total energy density of the Universe is in the form of the mysterious dark energy. While it dominates the dynamics of the Universe on the large scale, dark matter (matter whose existence has been inferred only through its gravity) clearly influences the galactic scale dynamics. Its abundance is given in terms of the density parameter $\Omega_{\text{dm}} = \rho_{\text{dm}}/\rho_c$, where ρ_c is the critical energy density. For the “concordance” flat Λ CDM model we have total matter $\Omega_{\text{m}} = 0.3$; in turn from emission and absorption of photons visible matter is roughly $\Omega_{\text{vis}} \simeq 0.04$ and it gives us that dark matter amounts to $\Omega_{\text{dm}} \simeq 0.26$.

The rotation curves of spiral galaxies give us the strongest evidence for dark matter. This dark matter in its 80% is in some form cold and nonbaryonic. Therefore, we can identify only 4% of the total matter content in the present universe. If we do not postulate the existence of dark matter it is not possible to explain why at large distance r from the centre of a given galaxy, we would find circular velocity $v_c^2 \simeq GM_{\text{vis}}/r$, since visible matter is concentrated around its centre. For the observational results of rotation curves see Ref. [4]. However, observations show that v_c is independent of r at large distances ($v_c \sim 200 \text{ km s}^{-1}$ is its typical value). From numerical simulations of the halo formation we obtain density profiles for both small and large values $\rho_{\text{halo}} \propto r^{-\alpha}$ with $\alpha \in (1; 1.5)$ for small and $\alpha = 3$ for large distances [5, 6].

There are two main candidates for cold dark matter, namely the axion and neutralino [7, 8]. In this paper we concentrate on the axion. Axions were originally proposed to solve the strong CP problem in the QCD. If axions have low mass, thus preventing other decay modes, axion theories predict that the universe would be filled with cold Bose-Einstein condensate of primordial axions. The axions in this condensate are always nonrelativistic and if the mass is about 10^{-3} eV it would plausibly explain the dark matter problem. There are prospects for direct experimental detection of axion. The Axion Dark Matter Experiment (ADMX) [9] searches for weakly interacting axions in the dark matter halo of our galaxy. Unfortunately, studies of axion dark matter are not sufficiently sensitive to probe the mass regions where axions would be expected. We investigate the possibility of explanation of flat velocity curves of spiral galaxies in terms of axion condensate which can be present in the Universe since the Peccei-Quinn phase transition.

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In our investigation we use the Gross-Pitaevski equation in an expanding FRW universe

$$i\hbar \left(\frac{\partial}{\partial t} + \frac{3}{2} \frac{\dot{a}(t)}{a(t)} \right) \phi(\vec{r}, t) = \left(-\frac{\hbar^2}{2m} \frac{1}{a^2(t)} \nabla^2 + U(\vec{r}) + g^2 |\phi(\vec{r}, t)|^2 \right) \phi(\vec{r}, t), \quad (1)$$

where $U(\vec{r})$ is the external potential, g^2 is the coupling constant between axions and $a(t)$ is the scale factor. Here we describe the condensate by one particle wave function $\phi(\vec{r}, t)$. The circulation in condensate is expressed by [10]

$$\Gamma = \oint_C \vec{v} \cdot d\vec{r} = \frac{\hbar}{m} 2\pi l, \quad (2)$$

where l is an integer called topological charge. C denotes any contour around a vortex. When there is no vortex, $l = 0$ and the circulation vanishes. When the condensate is inside a rotating environment the vortex is formed. In the interacting condensate vortices with $l > 1$ are unstable and decay to the vortices with $l = 1$. So in a realistic situation we have a net of elementary vortices ($l = 1$) which are stable. Such nets of vortices are observed in laboratories [11].

We study the possibility that a galactic halo is just such a vortex. We suppose that such a mechanism can be realised in the early universe. Namely, in the presence of global rotation, the proposed mechanism yields a huge amount of small vortices whose topological charges equal 1. In this paper we consider a singular vortex in the axion condensate.

The question whether rotation is an attribute of the Universe as a whole has been investigated since classical works of Lanczos [12], Gamov [13], Godel [14], Hawking [15] and recently by Chapline and Mazur [18]. When compared with the CMB anisotropies, the effects of rotation should not be big today [16]. Barrow et al. [17] showed that the cosmic vorticity depends strongly on the cosmological model and for a flat universe the bound on vorticity relative to the Hubble parameter at present is $\omega/H = 2 \times 10^{-5}$.

II. FREE AXION CONDENSATE

First, we must solve equation (1) to obtain the density distribution in the vortex. We know that the interaction between axions is very weak [19], so in the first approximation we can omit nonlinear term and external potential in equation (1). Then we consider, in fact, a free particle equation

$$i\hbar \left(\frac{\partial}{\partial t} + \frac{3}{2} \frac{\dot{a}(t)}{a(t)} \right) \phi(\vec{r}, t) = -\frac{\hbar^2}{2m} \frac{1}{a^2(t)} \left[\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} \right] \phi(\vec{r}, t). \quad (3)$$

In case of the de Sitter evolution of universe ($a(t) = \exp(Ht)$) the solution has the form

$$\phi(\vec{r}, t) = C \cdot \exp \left\{ -\frac{3}{2} Ht + \frac{1}{2} \frac{i\mu}{H\hbar} e^{-2Ht} \right\} j_l(\sqrt{\lambda} r) Y_l^n(\theta, \varphi), \quad (4)$$

where $\lambda = 2m\mu/\hbar^2$. Here $j_l(x)$ is the spherical Bessel function related to the ordinary one with

$$j_l(x) = \sqrt{\frac{\pi}{2x}} J_{l+\frac{1}{2}}(x). \quad (5)$$

and Y_l^n are the standard spherical harmonics.

Solution (4) is non-normalisable. To obtain a normalisable solution we must introduce proper boundary conditions, for example $\phi(r \geq \bar{r}) = 0$ for some \bar{r} . But it is not natural in our case, as our Universe is filled homogeneously by the axion condensate. Free axions should thus be described approximately by a non-normalisable waves with defined energies. Accordingly, our solution describes an axion condensate with defined nonquantized energy.

We neglected the interaction between axions in the above, but the gravitational interaction on the global scale may be important. In the next section we introduce a mean potential to better describe the gravitational interactions of axions.

Our main idea is that small vortices created in the early universe can grow during expansion of the universe. At the early stage, the vortex is a quantum object but during expansion it becomes a classical object. We assume that during this transition the shape of the vortex is conserved. To calculate how much the vortex has grown we need to know when its creation took place. To obtain an upper bound for this value we use the relation [19]

$$m_a \simeq 6\mu \text{ eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right), \quad (6)$$

where f_a is the energy of $U_{PQ}(1)$ symmetry breaking and creation of axions. There is some evidence that $m_a = 10^{-3}\text{eV}$ [20], for this value of energy is

$$f_a \simeq 6 \cdot 10^9 \text{GeV}. \quad (7)$$

Taking the scale factor $a_0 = 1$ (at the time of the creation of condensate) we have today

$$a = \frac{f_a}{3.7\text{K}} = \frac{6 \cdot 10^9 \text{GeV}}{3.2 \cdot 10^{-13} \text{GeV}} \simeq 2 \cdot 10^{22}. \quad (8)$$

So we see that the vortex cannot have grown more than $\sim 10^{22}$ times. Now we need to calculate the value of the scale factor a inferred from our solutions. Let us examine first the case without the vortex $l = 0$. The solution is of the form

$$\phi(\vec{r}, t) = C(t) \cdot \frac{\sin(\sqrt{\lambda}r)}{\sqrt{\lambda}r}. \quad (9)$$

This solution describes a spherically symmetrical halo with the density distribution $\rho(r, t) \sim |\phi(\vec{r}, t)|^2$. We must remember that the coordinate r is not a physical distance, which is $R = a \cdot r$. For a spherical distribution we calculate the velocity rotation curve from the relation

$$v(R) = \sqrt{\frac{G\mathcal{M}(R)}{R}}, \quad (10)$$

where $\mathcal{M}(R)$ is the mass function and is expressed as

$$\mathcal{M}(R) = 4\pi \int_0^R R'^2 \rho(R') dR'. \quad (11)$$

Figure (1) shows the velocity curve for axion condensate with assumed $\sqrt{\lambda}/a = 0.2 \text{ kpc}$.

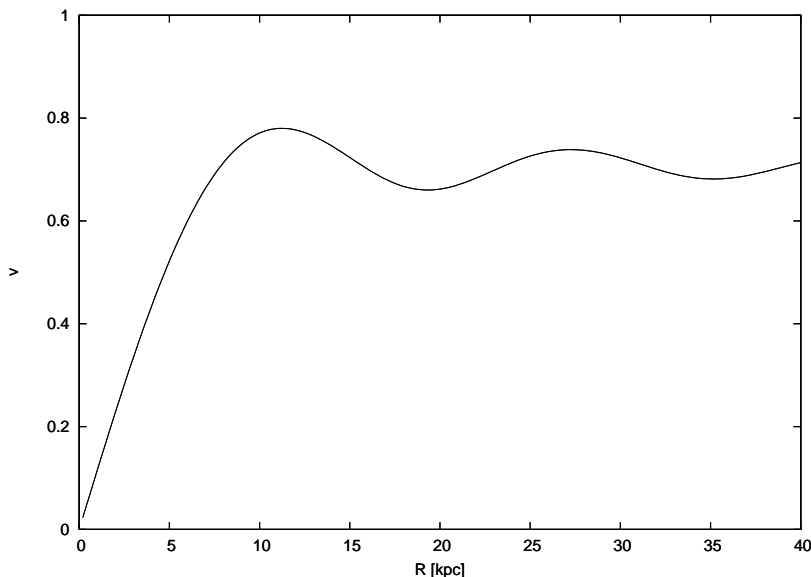


FIG. 1: The contribution to the galactic velocity curve given by the axion condensate with assumed $\sqrt{\lambda}/a = 0.2 \text{ kpc}$.

This result describes the observed galaxy velocity curves quite well – we can see the characteristic change to a plateau in the rotation curve. For further analysis we choose the point of transition to plateau in the rotation curve as our observable, which is estimated to take place for $R_{\text{trans}} \simeq 10 \text{ kpc}$ (R is the physical value, measured today). From our theory we have (for $l = 0$)

$$R_{\text{trans}} \simeq 2 \frac{a}{\sqrt{\lambda}}. \quad (12)$$

Taking $\mu = \frac{mv^2}{2}$ we have

$$\sqrt{\lambda} = \frac{mc^2}{\hbar c} \left(\frac{v}{c} \right) \simeq 2 \cdot 10^{20} \left(\frac{v}{c} \right) \text{pc}^{-1}. \quad (13)$$

Taking (12) and (13) for given R_{trans} we obtain the relation

$$a \simeq \left(\frac{v}{c} \right) \cdot 10^{24}. \quad (14)$$

When we compare this relation with the value in (8) we obtain the restriction for axion velocities

$$\left(\frac{v}{c} \right) < 10^{-2}. \quad (15)$$

It is naturally satisfied by axions as gravitationally induced velocities of axions are of the order

$$\left(\frac{v}{c} \right)_{\text{grav}} \sim 10^{-3}. \quad (16)$$

Comparing relations (8) and (14), we can see that in this model, the creation of vortices took place immediately after creation of axions.

Very similar results can be obtained considering the vortex solution with $l = 1$. The velocity curves also exhibit a plateau, like the solution with $l = 0$. The only difference is that we now have the angle dependence $\rho \sim \cos^2 \theta$. This produces, like for each vertex solution, distortions from the spherical shape of a dark matter halo.

Until now we have considered non-normalisable one-particle states. This state can describe the ensemble of axions only if they have the same energy. In fact, axions can have general distributions of energy. The dispersion of velocities of axions is very small [19] but it can produce normalised states. Thus, we construct the condensate as a superposition of one particle states

$$\Psi(\vec{r}, t) = \int_{-\infty}^{\infty} dk f(k) \phi_k(\vec{r}, t) = C Y_l^n(\theta, \varphi) e^{-\frac{3}{2}Ht} \int_{-\infty}^{\infty} dk f(k) \exp\left(\frac{1}{4} \frac{i\hbar k^2}{Hm} e^{-2Ht}\right) j_l(kr). \quad (17)$$

It is natural to choose $f(k)$ as the Gaussian distribution, but in this case we cannot compute the integral explicitly. That is why we choose the function

$$f(k) = k^2 e^{-(k/k_0)^2}, \quad (18)$$

which could be considered a shifted Gauss function and leads to analytical expressions. Unfortunately, dispersion and shift are now coupled. Assuming this distribution we obtain a normalised wave function

$$\Psi(\vec{r}, t) = \frac{k_0^{3/2}}{2\sqrt{\pi}a^3\sqrt{\frac{\pi}{2}}} \frac{e^{-(r/r_0)^2}}{\left[1 + \left(\frac{\hbar k_0^2}{8mHa^2}\right)^2\right]^{3/4}}, \quad (19)$$

where

$$r_0 = \frac{1}{k_0} \sqrt{1 + \left(\frac{\hbar k_0^2}{8mHa^2}\right)^2}. \quad (20)$$

The velocity curve for this model with $ar_0 = 10$ kpc is shown in Figure 2. As we can see in this figure, after taking a normalisable superposition of states, the velocity curve no longer has a plateau. For higher distances the velocity starts to decrease.

Let us now investigate the length scale of axion distribution. Taking

$$\frac{R}{ar_0} = 1, \quad (21)$$

for $R = 10$ kpc, we obtain the relation

$$a = 1.6 \left(\frac{v}{c} \right) \cdot 10^{24}, \quad (22)$$

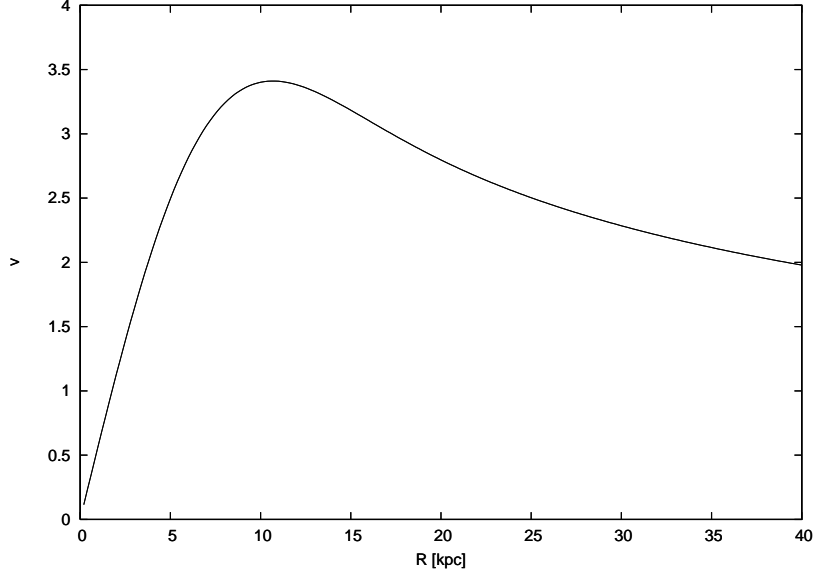


FIG. 2: The contribution to the galactic velocity curve given by the axion condensate with $ar_0 = 10$ kpc.

which is equivalent to relation (14). The case of interest to us is $l = 1$ because it is a stable vortex configuration. We then take

$$Y_1^1(\theta, \varphi) = \frac{1}{\sqrt{2\pi}} \frac{\sqrt{6}}{2} \cos \theta e^{i\varphi}, \quad (23)$$

and the solution is

$$\phi(\vec{r}, t) = C(t) \left(\frac{\sin(\sqrt{\lambda}r)}{(\sqrt{\lambda}r)^2} - \frac{\cos(\sqrt{\lambda}r)}{\sqrt{\lambda}r} \right) \cos(\theta) e^{i\varphi}. \quad (24)$$

Note, that in the case considered, the density distribution strongly depends on the angle θ , namely $\rho \propto \cos^2 \theta$. Next, we construct a superposition of one-particle states using the distribution function

$$f(k) = k e^{-\frac{1}{2}(k/k_0)^2}. \quad (25)$$

This leads to the expression for the wave function

$$\Psi(\vec{R}, t) = C \frac{1}{a^{3/2}} \frac{1}{R^2} \cdot \left[\operatorname{erf} \left(\frac{1}{2} \frac{R}{\xi_1 \sqrt{1+i\xi_2}} \right) - \frac{1}{\sqrt{\pi}} \frac{R}{\xi_1 \sqrt{1+i\xi_2}} \exp \left(-\frac{1}{4} \frac{R^2}{\xi_1^2 (1+i\xi_2)} \right) \right] \cos \theta e^{i\varphi}, \quad (26)$$

where

$$\xi_1 = \frac{a}{\sqrt{2}} \frac{1}{k_0} = \frac{a}{\sqrt{2}} \frac{\hbar c}{mc^2} \left(\frac{v_0}{c} \right)^{-1} \quad (27)$$

$$\xi_2 = \frac{\hbar}{4Hm} \frac{1}{\xi_1^2}. \quad (28)$$

Choosing the realistic values of parameters $m = 10^{-3}$ eV, $H = 70$ km s $^{-1}$ Mpc $^{-1}$, $a = 10^{21}$, $v_0/c = 10^{-3}$ we obtain $\xi_1 = 4.5$ kpc and $\xi_2 = 3.4 \cdot 10^{-13}$. Using relation (10) we can now compute the velocity curve. Because relation (10) is valid for spherical distributions we compute only the mean velocity curve. This result is shown in Figure 3.

III. GRAVITATIONAL INTERACTION OF AXIONS

Up to now we have neglected the interaction of axions. But in fact, the dispersion of velocities is a result of interaction. This effect was studied in the previous section. Now we include the gravitational interaction by means

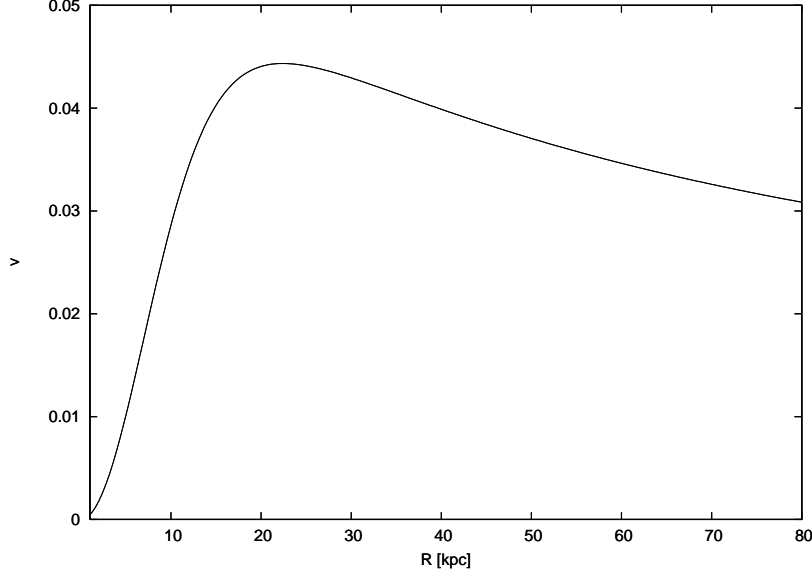


FIG. 3: The contribution to the galactic velocity curve given by the axion condensate with $\xi_1 = 4.5$ kpc and $\xi_2 = 3.4 \cdot 10^{-13}$.

of an effective potential. We introduce the average density distribution of axions $\langle \rho \rangle = \langle \rho \rangle_0 / a^3$ which includes both condensed and noncondensed parts. Since we work in the Newtonian approximation, we solve the Poisson equation

$$\frac{1}{a^2(t)} \nabla^2 V = 4\pi G \frac{\langle \rho \rangle_0}{a^3(t)}, \quad (29)$$

which in this case leads to the effective potential

$$U(\vec{r}) = \frac{1}{2} \frac{m\omega^2 r^2}{a(t)}, \quad (30)$$

where

$$\omega^2 = \frac{4}{3} \pi G \langle \rho \rangle_0. \quad (31)$$

Now we have to solve the equation

$$i\hbar \left(\frac{\partial}{\partial t} + \frac{3}{2} \frac{\dot{a}(t)}{a(t)} \right) \phi(\vec{r}, t) = -\frac{\hbar^2}{2m} \frac{1}{a^2(t)} \left[\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} \right] \phi(\vec{r}, t) + \frac{1}{2} \frac{1}{a(t)} m\omega^2 r^2 \phi(\vec{r}, t). \quad (32)$$

This time it is not possible to separate variables as easily as it was done before, because the right hand side of the equation explicitly depends on time. Relative contributions of the right hand side components depend on the factor $1/a$. So we can see that during the evolution, the interaction term becomes more and more important. We cannot find the general solution of equation (32) but we can investigate the boundary solutions. Taking $a = 1$, we obtain

$$\phi(\vec{r}, a = 1) = \frac{C}{r_0} e^{-\frac{1}{2} \left(\frac{r}{r_0} \right)^2} \left(\frac{r}{r_0} \right)^{(l)} {}_1F_1 \left[-n_r, l + \frac{3}{2}; \left(\frac{r}{r_0} \right)^2 \right] Y_l^n(\theta, \varphi), \quad (33)$$

where $r_0^{-1} = \sqrt{m\omega/\hbar}$, and ${}_1F_1$ is the confluent hypergeometric function of the first kind. The value of the chemical potential is quantised as follows

$$\mu = \hbar\omega \left(2n_r + l + \frac{3}{2} \right). \quad (34)$$

IV. SUMMARY

We have considered axion condensate as a candidate for dark matter in galactic halos. Condensates in laboratories are known to exhibit vortices induced by rotation of the environment. Such vortices are unstable and decay into states with the lowest angular momentum. We prolong this picture onto the entire Universe whose global rotation might give rise to the rotation of particular galaxies.

Summing-up, we firstly considered non-normalisable states with $l = 0$ and $l = 1$ (vortex). It was shown that these solutions leads to the flat velocity curves characterised by oscillations in a plateau region. Next, we introduced the axion velocity dispersion to obtain well normalised states. Such a velocity dispersion might come from gravitational interactions of axions. In this case we calculated contributions to velocity curves which, after reaching a maximum, slowly decreases. Finally we mentioned the problem of axion interaction, which requires a further numerical analysis.

Beside the problem of velocity curves the scenario considered can explain also the galaxy mass problem. As we know masses of the spiral galaxies are similar and equal to $\sim 10^{11} M_{\odot}$. This can be naturally explained since all vortices are similar, what is directly a quantum effect.

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- [1] D. N. Spergel *et al.*, arXiv:astro-ph/0603449
 - [2] A. Riess *et al.*, *Astron. J.* **116**, 1009 (1998), arXiv:astro-ph/9805201
 - [3] S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999), arXiv:astro-ph/9812133
 - [4] M. Persic, P. Salucci and F. Stel, *Mon. Not. Roy. Astron. Soc.* **281**, 27 (1996), arXiv:astro-ph/9506004
 - [5] B. Moore *et al.*, *Mon. Not. Roy. Astron. Soc.* **310**, 1147 (1999)
 - [6] J. F. Navarro, C. S. Frenk, and S. D. M. White, *Astrophys. J.* **490**, 493 (1997), arXiv:astro-ph/9611107
 - [7] G. Lazarides, arXiv:hep-ph/0601016
 - [8] P. Gondolo, *NATO Sci. Ser. II* **187**, 279 (2005), arXiv:astro-ph/0403064
 - [9] L. D. Duffy *et al.*, *Phys. Rev. D* **74**, 012006 (2006), arXiv:astro-ph/0603108
 - [10] S. Ghosh, *Phase Transitions*, **77**, 625 (2004), arXiv:cond-mat/0405589
 - [11] F. Chevy, K. Madison, V. Bretin, and J. Dalibard, arXiv:cond-mat/0104218
 - [12] K. Lanczos, *Z. Phys.* **21**, 73 (1924)
 - [13] G. Gamov, *Nature* **158**, 549 (1946)
 - [14] K. Godel, *Rev. Mod. Phys.* **21**, 447 (1949)
 - [15] S. W. Hawking, *Mon. Not. Roy. Astron. Soc.* **142**, 129 (1969)
 - [16] J. Silk, *Mon. Not. Roy. Astron. Soc.* **147**, 13 (1970)
 - [17] J. D. Barrow, R. Juszkiewicz, and D. Sonoda, *Mon. Not. Roy. Astron. Soc.* **213**, 917 (1985)
 - [18] G. Chapline and P. O. Mazur, *AIP Conf. Proc.* **822**, 160 (2006)
 - [19] P. Sikivie, arXiv:astro-ph/0610440
 - [20] E. Zavattini *et al.*, *Phys. Rev. Lett.* **96**, 110406 (2006), arXiv:hep-ex/0507107